Experimental verification that photons carry momentum using the Compton effect

by Martin Mueller – The Open University, February 20, 2023

Introduction

The aim of the experiment was to verify that photons emerge from electromagnetic radiation and carry the particle-like property of momentum. Based on Max Planck's assumption that the energy of electromagnetic radiation is quantized, Albert Einstein postulated in 1905 in explaining the photoelectric effect that it consists of energy quanta that propagate as such through space (The Open University, 2022a). In the early 1920s, Arthur Holly Compton conducted experiments on the scattering of X-rays by electrons and was able to establish that energy quanta, later called photons, have another particle-like property, namely momentum. He was able to derive a theory from the existing laws of conservation of relativistic energy and momentum that predicted the results of his experiments very well (The Open University, 2022b).

The fact that photons carry momentum led to the counterintuitive understanding of particle-wave duality, which is one of the cornerstones of quantum theory and formed the basis for its further evolution in the years that followed.

The question to be answered is: Can the theoretical predictions of the Compton formula, which is based on the laws of conservation of energy and momentum, be confirmed by an experiment similar to the one carried out by Compton in the 1920s?

Methods and results

Theory

The theory to be tested states that electromagnetic radiation consists of photons which, when they hit electrons at rest, transfer momentum, lose energy and are scattered at angles described by relativistic energy and momentum conservation laws. This assumption is depicted in Figure 1.

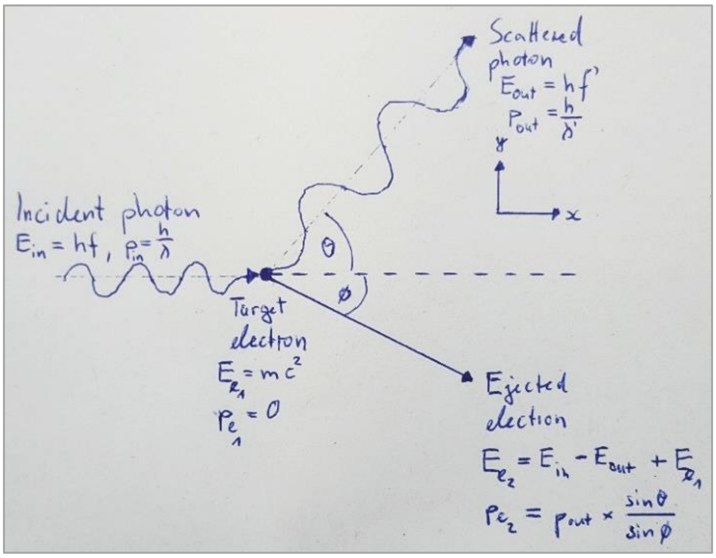


Figure 1: Schematic representation of the application of the laws of conservation of energy and momentum to determine the energies of the scattered photons E_{out} and their corresponding scattering angles ϑ, where E_{in} are the X-ray energies incident on the electrons, E_e are the energies of the electrons, p_{in} and p_{out} are the momenta of the photons, p_e are the momenta of the electrons, m is the rest mass of the target electrons, c is the speed of light, h is Planck's constant and φ is the scattering angle of the electrons. It is pictured that the wavelengths λ of the scattered photons are larger than those of the incident photons, equivalent to the expected energy losses.

The relationship between the reduced energies E_{out} of the scattered photons and their scattered angle θ is describe by the Compton formula

$$E_{out} = \frac{E_{in}}{1 + \frac{E_{in}}{m_e c^2}[1 - \cos\theta]} \qquad (1)$$

which can be derived from three relationships and principles (The Open University, 2022c),

i) Energy – momentum relationship for relativistic particles with no mass

$$p = \frac{E}{c} \qquad (2)$$

ii) Conservation of momentum

$$p_x: \quad \frac{E_{in}}{c} = \frac{E_{out}}{c}\cos\theta + p_e\cos\varphi \qquad (3.1)$$

$$p_y: \quad 0 = \frac{E_{out}}{c}\sin\theta - p_e\sin\varphi \qquad (3.2)$$

iii) Conservation of relativistic energy

$$E_{in} + m_e c^2 = E_{out} + E_e \qquad (4).$$

Experimental equipment

The key instrument used in the experiment was an X-ray apparatus on the campus of the Open University in Milton Keynes, United Kingdom. It could be operated remotely and thus was suitable for team work of students at different locations. As shown in Figure 2, it consisted of a vacuumised tube in which electrons were thermally emitted and then accelerated from the cathode to the molybdenum anode, where electrons were ejected and, after decaying into lower shells, X-rays were produced with energies mainly of 17.4 keV and 19.6 keV. A collimator focused the X-rays on the target, which was made of perspex and could be inclined. A detector, which could be tilted between 30° and 150°, made it possible to detect the scattered photons at specific angles, and a pulse height analyser sorted the various measured pulses into channels (representing energies) for further data processing (The Open University, 2022d).

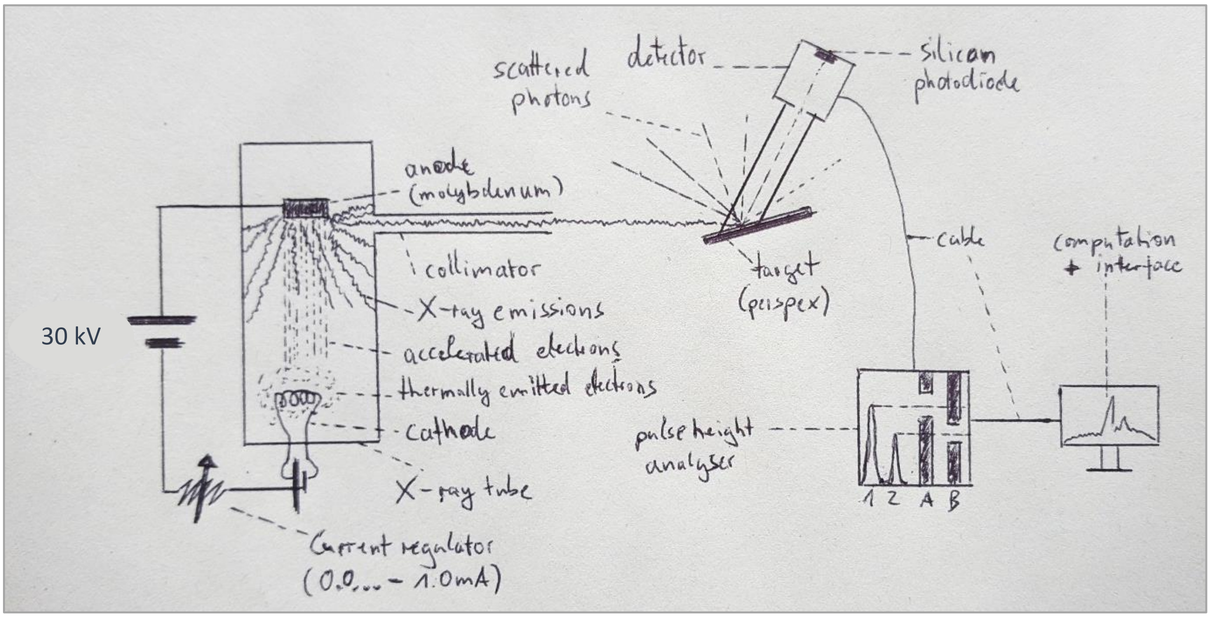


Figure 2: Diagram of the main components of the X-ray apparatus and their use and effects in the experiment

The whole experiment was conducted under the guidance of the Open University study material and can be divided into 5 consecutive main steps with corresponding intermediate results after their completion. The first activity was a team effort by two students, namely Matthew West and the author of this report, Martin Mueller.

Step 1: Creating an experimental plan and carrying out the experiment in a team of two students

The experiment was planned in detail by the team, taking into account the following time and technical constraints:

- The X-ray apparatus was only available during a time slot of 90 minutes.
- The apparatus was not provided in a calibrated state.
- The optimum voltage to generate the X-rays was recommended to be 30 kV.
- The current for the X-ray intensity in the calibration was recommended to be between 0.1 and 0.25 mA.
- The apparatus' detector could only move between 25° and 150°.
- The energy window, i.e. the resolution of the pulse height analyser could be selected by choosing the number of channels.
- The angular resolution could not be set.
- The angle of the target was recommended to be -90° for calibration and 20° for the scattering.
- The roles and tasks should rotate during the experiment.

In order to achieve maximum results, after the team exchanged views in the forum and in an online meeting, a plan as shown in Table 1 was jointly created (only columns 1-8 are from the planning stage). The first three scans were needed to find a suitable current for the calibration scan so that the detector would not become saturated. To achieve the largest possible number of scans, the longest possible scan time and the largest possible angular range, the team decided to perform scans between 30° and 150° in 15° increments and with a scan time of 5 minutes each. As a compromise between statistical error and energy resolution, and after a review of the study material and student forum entries, it was decided to use 512 channels.

1	2	3	4	5	6	7	8	9	10	11
Target angle (°)	Detector angle (°)	Voltage (kV)	Current (mA)	Count time (sec.)	Handling time (min)	Person doing scanning	Person keeping records	Prediction (K_α)	Measured channel (1 d.p.)	Calculated Channel Error (1 d.p.)
-90	0	30	0.10	20	NA	Martin	Matthew	NA	NA	NA
-90	0	30	0.15	20	NA	Martin	Matthew	NA	NA	NA
-90	0	30	0.20	20	NA	Martin	Matthew	NA	NA	NA
-90	0	30	0.10	300	2	Martin	Matthew	NA	K_α: 343.2 K_β: 384.0	K_α: 0.1 K_β: 0.2
20	30	30	1.00	300	2	Martin	Matthew	341.7	K_α: 342.4	0.1
20	45	30	1.00	300	2	Martin	Matthew	340.0	K_α: 340.5	0.2
20	60	30	1.00	300	2	Martin	Matthew	337.8	K_α: 338.8	0.2
20	75	30	1.00	300	2	Martin	Matthew	335.3	K_α: 336.3	0.2
20	90	30	1.00	300	2	Matthew	Martin	332.6	K_α: 333.8	0.3
20	105	30	1.00	300	2	Matthew	Martin	329.9	K_α: 331.4	0.4
20	120	30	1.00	300	2	Matthew	Martin	327.5	K_α: 328.4	0.4
20	135	30	1.00	300	2	Matthew	Martin	325.5	K_α: 325.5	0.3
20	150	30	1.00	300	2	Matthew	Martin	323.9	K_α: 324.5	0.4

Table 1: Table summarizing the results of planning the experiment (columns 1-8) which also served as execution plan. The orange shaded rows were calibration steps. The channels in column 9 are the calibrated results when entering the detector angle into the Compton formula. Column 10 and 11 show the measured channels and calculated errors as identified in the subsequent step (step 2).

The experiment went smoothly and all scans could be performed as planned and with clear peaks for the X-ray energies of 17.4 keV (K_α) and 19.6 keV (K_β) (see example in Figure 3). During the calibration phase it turned out that in the team's setup a current of 0.1 mA was optimal to reach about the recommended 200 photons per second. The raw data of all 10 full scans were downloaded for subsequent data processing.

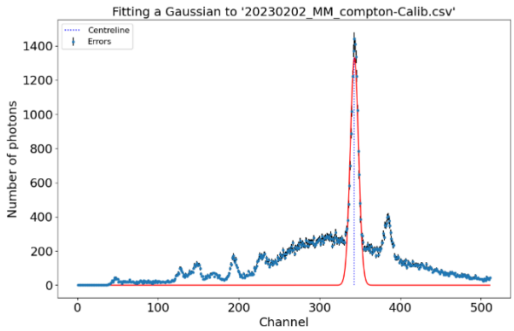
Figure 3: Screenshot of the calibration scan in the live experiment. The greatest peak originates from photons with an energy of 17.4 keV (K_α – decay of electrons in Molybdenum) and the second greatest peak originates from photons with an energy of 19.6 keV (K_β - decay of electrons in Molybdenum).

Step 2: Identifying the energies of the unscattered and scattered photons and their errors

The emission of X-rays due to the decay of electrons into the lower shells of the molybdenum atoms in the X-ray tube is random, as is the incidence of photons on the electrons in the target. The errors of the random processes in the experiment were considered using the standard deviation $\sqrt{N}$, where N were the counts of photons per channel. Further on, due to the limited scanning time and energy resolution in the experiment, the photon count obtained per channel was not always the most accurate result. To overcome the time and resolution limitations a non-linear least squares regression based on a Gaussian function was applied to the data. This was realised with the help of Python's *curve_fit* function from the *Scipy* library. A Python program was then created to be applied to both the calibration scan (see Figure 4a) and the nine scatter scans (see examples in Figure 4b) providing the statistically most probable channels (x0) and their errors.

```
Estimated peak height: 1435
Estimated peak position: 343
Estimated FWHM: 10

fit parameters with error estimates
********************************************
A     =  1331 +/-  43.43
x0    =  343.2 +/-  0.1402
sigma =  5.131 +/-  0.1413
********************************************
```




```
Estimated peak height: 400
Estimated peak position: 385
Estimated FWHM: 10

fit parameters with error estimates
********************************************
A     =  365.8 +/-  8.59
x0    =  384 +/-  0.1961
sigma =  7.754 +/-  0.2725
********************************************
```

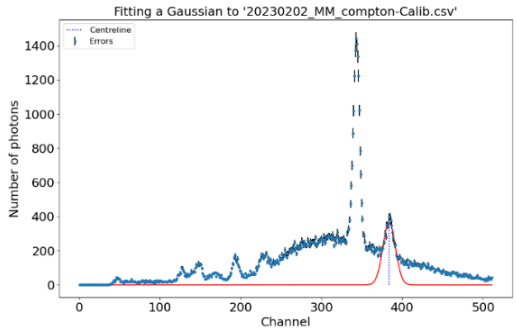
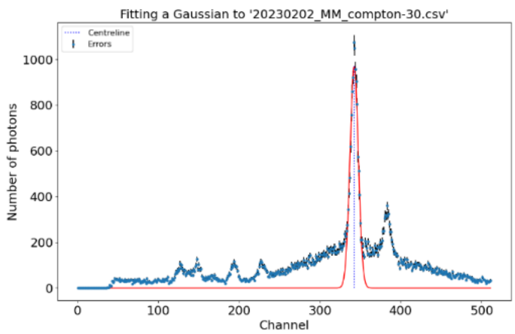


Figure 4a: Gaussian fit to the calibration scan in order to find the channels for K_α and K_β. Three estimated values (peak height A, peak position x0, FWHM) were passed to the curve_fit function. In the left program run a Gaussian fit was applied for the peak to K_α, in the right program run for the peak to K_β.

```
Estimated peak height: 1070
Estimated peak position: 343
Estimated FWHM: 10

fit parameters with error estimates
**********************************************
*
A     =  969.5 +/-  30.52
x0    =  342.4 +/-  0.1373
sigma =  5.148 +/-  0.1376
**********************************************
*
```




```
Estimated peak height: 545
Estimated peak position: 326
Estimated FWHM: 15

fit parameters with error estimates
**********************************************
*
A     =  475.9 +/-  18.23
x0    =  324.5 +/-  0.3517
sigma =  9.761 +/-  0.4434
**********************************************
*
```

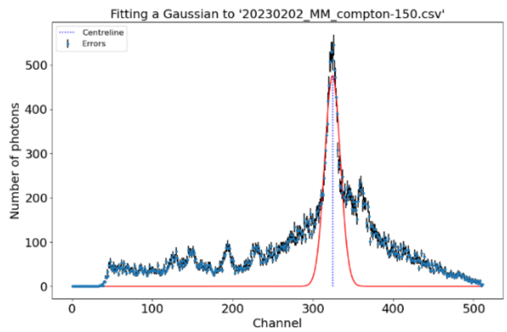


Figure 4b: Gaussian fit to two of the nine scatter scans in order to find the appropriate channels for K_α. Three estimated values (peak height A, peak position x0, FWHM) were passed to the curve_fit function. The left program run applies a Gaussian fit to the scattered photons from incident energy K_α at scattering angle 30° and the right program run applies a Gaussian fit to the scattered photons from incident energy K_α at scattering angle 150°.

The two channels of the two calibration energies including their errors were noted to be applied to the subsequent calibration step and the nine channels including their errors resulting from K_α in the nine scattering scans were saved to a .csv file to be used for the final validation of the experiment. For information purposes, all identified channels and their calculated errors are also listed in columns 10 and 11 in Table 1.

Step 3: Calibrating the data

Since the pulse height analyser of the apparatus assigned photon counts to channels, however without indicating their energies, it was necessary to calibrate the data, i.e. to find a conversion of the channel numbers ch_{out} into energies $E_{out\_Experiment}$. For this purpose, the detected channels ch_K of the unscattered X-ray energies E_K with K_α at 17.4 keV and K_β at 19.6 keV as obtained in the calibration scan were taken to be used in the formula

$$E_{out_{Experiment}} = E_{K_\beta} + \left(ch_{out} - ch_{K_\beta}\right) \times \frac{\left(E_{K_\alpha} - E_{K_\beta}\right)}{\left(ch_{K_\alpha} - ch_{K_\beta}\right)} \qquad (5).$$

Step 4: Developing a model incorporating all major uncertainties that allows the comparison between the experimental data and the theoretical predictions

As outlined in step 1, only 9 measurements could be carried out, which were further subject to some uncertainties due to technical and time constraints. Given this background, in order to obtain a solid basis for comparing the experimental results with the theory, both the data from the experiment and the theory were transformed into a linear relationship of the form

$$y = \quad m \quad \times \quad x \quad + \quad c \qquad (6)$$

For this, the Compton formula (see equation 1) was rearranged to give

$$\frac{E_{in}}{E_{out}} = \frac{E_{in}}{m_e c^2} \times (1 - \cos\theta) + 1 \qquad (7),$$

whereas the following form was finally used

$$\frac{1}{E_{out}} = \frac{1}{m_e c^2} \times (1 - \cos\theta) + \frac{1}{E_{in}} \qquad (8).$$

In this form $\frac{1}{E_{out}}$ corresponded to y, $\frac{1}{m_e c^2}$ corresponded to m, $(1 - \cos\theta)$ corresponded to x and $\frac{1}{E_{in}}$ corresponded to c. Accordingly, for the experimental data $\frac{1}{E_{out_{Experiment}}}$ corresponded to y and $(1 - \cos\theta)$ corresponded to x.

In order to find the overall errors for the experimental data a linear least squares regression fit was performed using Python's orthogonal distance regression (ODR) from the Scipy library. This fitting algorithm was chosen because errors in x as well as in y had to be taken into account.

As for errors in x, information was given in the forums by the module specialists of the Open University that the error in the angles was 3% at the maximum. Since x = 1-cos(θ), the errors in x were calculated according to the rules for errors in cosine functions (The Open University, 2022e), so that

$$x_{error} = \pm \frac{\cos(\theta \times 1.03) - \cos(\theta \times 0.97)}{2} \qquad (9).$$

The errors in y needed to consider a combination of errors since $E_{out\_Experiment}$ was calculated using several quantities, as can be seen from equation 5. The values for E_{K_α} = 17.4 keV and E_{K_β} = 19.6 keV, which were produced by the decay of electrons to lower energy levels in the molybdenum atom are quantized at high precision and thus their errors could be neglected. The individual errors for ch_{out}, ch_{K_α} and ch_{K_β} were gathered from step 2. Combining them required several operations, alternating between absolute and percentage errors (The Open University, 2022e), as follows:

1.1) Calculate the combined absolute error of $[ch_{out} - ch_{K_\beta}]$, i.e. $\sqrt{{ch_{out}}^2 + {ch_{K_\beta}}^2}$

1.2) Calculate the percentage error of result of step 1.1, i.e. $[Result\ of\ 1.1] \div \left(ch_{K_\beta} - ch_{out}\right)$

2.1) Calculate the combined absolute error of $[ch_{K_\alpha} - ch_{K_\beta}]$, i.e. $\sqrt{{ch_{K_\alpha}}^2 + {ch_{K_\beta}}^2}$

2.2) Calculate the percentage error of 2.1, i.e. $[Result\ of\ 2.1] \div \left(ch_{K_\beta} - ch_{K_\alpha}\right)$

3.) Calculate the combined percentage error of [1.2 / 2.2], i.e. $\sqrt{[Result\ of\ 1.2]^2 + [Result\ of\ 2.2]^2}$

4.) Calculate the final absolute error, i.e. $y_{error} = \pm\, y \times [Result\ of\ 3] \qquad (10).$

Step 5: Writing a Python program executing the calculations developed in steps 3 and 4 and providing a comparison between experimental data and theoretical prediction

To compare experimental data with the theoretical predictions a Python program was written, mainly executing the following operations:

- Read the channels and errors obtained from step 2
- Calibrate the data according to step 3
- Calculate the errors in x and y according to step 4
- Generate a linear best fit line for the data in the form y = mx + c and calculate errors for the gradient m and the y-intercept c using the Python function Scipy ODR according to step 4
- Generate a straight line based on the Compton formula in the form y = mx + c according to step 4, equation 8
- Plot the comparison including the error ranges (see Figure 6)

The numerical result of the Python program from step 5 is presented in Figure 5a. Figure 5b shows the values from the application of the Compton formula, which allows a direct comparison. The difference in the gradient m between the best fit line of the experimental data and the straight line predicted by the Compton formula is 0.73%, while the allowed error in m due the experimental setup and technical constraints is 15%. The difference in the y-intercept c between the experimental data and the predicted values is -0.28%, while the allowed error is 0.55%.

```
fit parameter 1-sigma error
*****************************
m = 1.2304e+13 +- 1.7999e+12
c = 3.5774e+14 +- 1.9571e+12
*****************************
```

Figure 5a: Screenshot of gradient m and y-intercept c of the best-fit line including errors as calculated and presented by the Python program developed in step 5

```
# The compton formula in the form y = mx +c is ...
# 1/E_in = 1/(m_e * c**2) * (1 - cos(zeta)) + 1/E_in

print (f'The gradient m derived from the Compton formula is .........{1/(m_e * c**2):.5g}')
print (f'The y-intercept c derived from the Compton formula is ..... {1/E_in:.5g}')
```

```
The gradient m derived from the Compton formula is .........1.2214e+13
The y-intercept c derived from the Compton formula is ..... 3.5875e+14
```

Figure 5b: Screenshot of a simple Python program calculating and presenting the values for gradient m and y-intercept c of by using the Compton formula

Discussion and interpretation

The result of this experiment and its comparison to the theoretical prediction by the Compton formula are visualized in Figure 6.

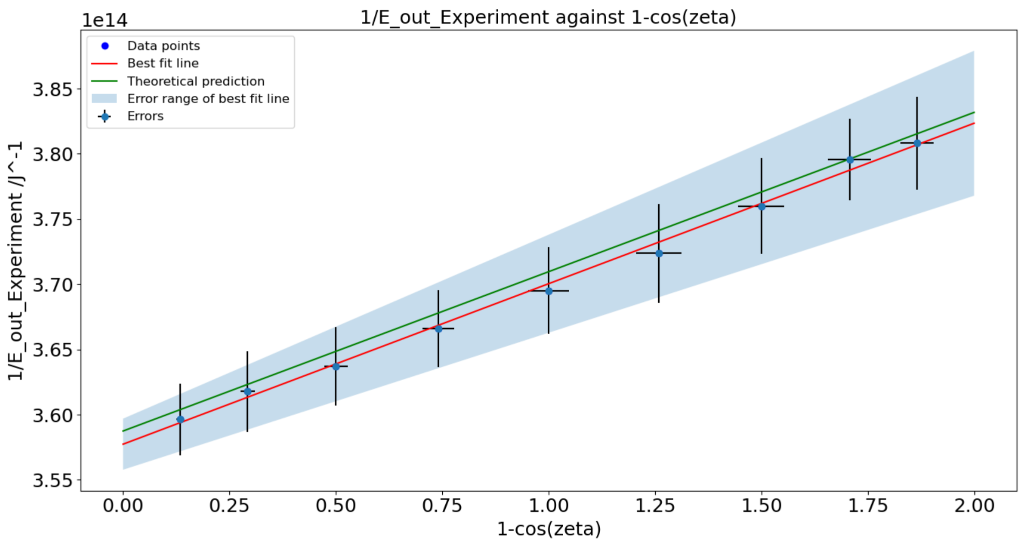


Figure 6: Screenshot of the Python plot showing the result of this experiment and its agreement with the theory. Data points represent scattered energies of the photons ($E_{out\_Experiment}$) at angles of 30°, 45°, 60°, 75°, 90°, 105°, 120°, 135° and 150°. The greater the angles were, the smaller were the measured energies with a linear relationship between $1/E_{out\_Experiment}$ and 1-cos(ϑ). This relationship is represented by a best-fit line calculated by linear least squares regression (red line). As can be seen, the line is almost parallel to the relationship derived from theory (green line) with only a small distance caused by different y-intercepts.

Even though not all errors might have been considered (e.g. uncertainties of the pulse height analyser and the detector) the differences in the gradients and the y-intercepts are well within the error ranges of this particular experiment as can been seen by the light blue shaded area. Since the Compton formula is solely derived by applying the relativistic energy and momentum conservation laws, it can be therefore deduced that photons not only possess quantised energies but also particle-like momenta. The experiment demonstrates vividly the non-classical wave-particle duality of electromagnetic radiation.

Conclusions

- The question to be answered was whether photons carry momentum and whether this could be found out by a Compton scattering experiment.
- In the experiment, nine clearly distinct scattering energies of photons at nine different angles could be detected, originating from the incident X-ray radiation of 17.4 keV.
- The key method of comparing the experimental data with the theory was to establish a linear relationship between the angles of detection of the photons and their energies.
- The comparison between the experimental data and the theoretical prediction confirmed the theory and thus the assumption that photons carry momentum.

References and acknowledgements

The Open University (2022a) ‘3.3 The photoelectric effect’. SXPS288-22J: Remote experiments in physics and space. Available at: https://learn2.open.ac.uk/mod/oucontent/view.php?id=1926151&section=3.3 (Accessed 13 February 2023).

The Open University (2022b) ‘3.5 What is Compton scattering?’. SXPS288-22J: Remote experiments in physics and space. Available at: https://learn2.open.ac.uk/mod/oucontent/view.php?id=1926151&section= 3.8 (Accessed 15 February 2023).

The Open University (2022c) ‘3.8 Deriving and using the Compton formula’. SXPS288-22J: Remote experiments in physics and space. Available at: https://learn2.open.ac.uk/mod/oucontent/view.php?id=1926151&section= 3.8 (Accessed 15 February 2023).

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